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Influence of transverse shear on stochastic instability of viscoelastic beam

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Abstract

The stochastic instability problem associated with an axially loaded Timochenko beam made of viscoelastic material is formulated. The beam is treated as Voigt–Kelvin body compressed by time-dependent deterministic and stochastic forces. By using the direct Liapunov method, bounds of the almost sure instability of beams as a function of retardation time, variance of the stochastic force, mode number, section shape factor and intensity of the deterministic component of axial loading, are obtained. Calculations are performed for the Gaussian process with a zero mean and variance σ^2 as well as for harmonic process with an amplitude A. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Stochastic instability; Timochenko beam; Instability regions; Gaussian process; Harmonic process; Section shape factor; Retardation time

1. Introduction

The dynamic stability of the continuous systems under time-dependent deterministic or stochastic loading has been studied by many authors for the last 40 years. The problem has been solved not only for simple elastic beams subjected to time-dependent force but also for arches, panels, plates and shells. In most of the papers the dissipation of energy was described by an external viscous model of damping. The "best" Liapunov functional suitable for studying the almost sure asymptotic stability of elastic beams and panels axially compressed by zero mean stationary ergodic forces, whose samples are continuous with probability one was introduced by Kozin (1972).

One of the first analyses of the dynamic stability of viscoelastic columns has been made by Plaut (1972), where the Liapunov method is used to determine the stability criteria. The stability problem associated with an Euler–Bernoulli beam made of an arbitrary linear viscoelastic material in presence of constant as well as periodic loads is analyzed by Shirahatti and Sinha (1994). The application of a finite time stability concept is shown for the constant loading case when the traditional stability criterion fails to make sense. For the case of a periodic loading, the stability diagrams are obtained by applying the Floquet theory.

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Tylikowski (1986) studied linear viscoelastic systems subjected to time-dependent deterministic or stochastic parametric excitations. By using the appropriate Liapunov functional, general sufficient conditions for the asymptotic stability, the almost sure asymptotic stability as well as the uniform stochastic stability are obtained.

The Voigt–Kelvin column with random initial geometrical imperfections is considered by Tylikowski (1990). The sufficient criterion for stability of viscoelastic composite columns via direct Liapunov method is obtained by the same author (1992). The influence of standard model parameters and eigenfrequencies on stability domains is shown.

Direct Liapunov method is used by Pavlović and Kozić (1993) for investigating stochastic stability of viscoelastic beams including the effect of rotatory inertia. By using suitable functionals obtained by the Parks–Pritchard method (1969), the sufficient conditions for almost sure asymptotic stability as a function of reduced retardation time and geometric parameters are derived.

It is well known that the effect of transverse shear can be significant when the cross-sectional dimensions of a beam are large in comparison to its length; the effect of rotatory inertia for higher modes is important as well.

The purpose of the present paper is the investigation of dynamic instability of viscoelastic beams subjected to time-dependent axial forces when transverse shear is taken into account. By using recent results of the stochastic processes theory we can apply the Liapunov method to obtain sufficient criteria for almost sure asymptotic instability in terms of retardation time, variance of the stochastic force, mode number, section shape factor and intensity of the deterministic component of axial loading. The principal contribution of this paper is that the influence of transverse shear on almost sure instability regions can sometimes be neglected, and in other cases such neglecting can be a serious error.

2. Problem formulation

Let us consider the viscoelastic beam of length ℓ, subjected to uniformly distributed stochastic axial time-dependent loading $F_a(\omega, T)$. The deflected shape of a typical element of a beam is shown in Fig. 1. F_T is the shear force due to bending, M is the bending moment, A is the cross-sectional area, I_x is the cross-section moment of inertia, w is the transverse displacement in the Y direction, T is the time and ρ is the mass density.

The dynamic equilibrium equations obtained from Fig. 1 are:

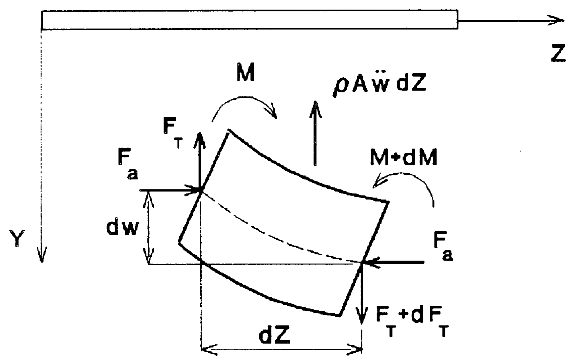

Fig. 1. Forces acting on a typical beam element.

$$\rho A \frac{\partial^2 w}{\partial T^2} - \frac{\partial F_T}{\partial Z} = 0, \tag{1}$$

$$F_T + F_a \frac{\partial w}{\partial Z} - \frac{\partial M}{\partial Z} = 0, \tag{2}$$

where shear deformation is taken into account, and rotatory inertia is neglected.

We will take into consideration a Voigt–Kelvin type of viscoelasticity where stress–strain relation has the form:

$$\sigma = E\left(\varepsilon + v\frac{d\varepsilon}{dT}\right), \tag{3}$$

where E is Young modulus and v is the retardation time.

Euler–Bernoulli relationship for small deflections, with respect to Eq. (3) gives:

$$\begin{aligned} F_T &= kAG\left(1 + v\frac{\partial}{\partial T}\right)\left(\frac{\partial w}{\partial Z} - \phi\right) - F_a\frac{\partial w}{\partial Z}, \\ M &= -EI_x\left(\frac{\partial \phi}{\partial Z} + v\frac{\partial^2 \phi}{\partial Z\,\partial T}\right), \end{aligned} \tag{4}$$

where k is section shape factor, G is shear modulus and ϕ is the shear angle of rotation of the cross-section. F_T and M can be eliminated from Eqs. (1), (2) and (4). The governing equations of motion for the beam are:

$$\begin{aligned} &\rho A\frac{\partial^2 w}{\partial T^2} - kAG\left(1 + v\frac{\partial}{\partial T}\right)\frac{\partial^2 w}{\partial Z^2} + kAG\left(1 + v\frac{\partial}{\partial T}\right)\frac{\partial \phi}{\partial Z} + \frac{\partial}{\partial Z}\left[F_a(\omega, T)\frac{\partial w}{\partial Z}\right] = 0, \\ &kAG\left(1 + v\frac{\partial}{\partial T}\right)\frac{\partial w}{\partial Z} - \left(kAG - EI_x\frac{\partial^2}{\partial Z^2}\right)\left(1 + v\frac{\partial}{\partial T}\right)\phi = 0. \end{aligned} \tag{5}$$

By applying differential operators

$$L_{10} = kAG - EI_x\frac{\partial^2}{\partial Z^2}, \qquad L_{20} = kAG\frac{\partial}{\partial Z} \tag{6}$$

to differential Eqs. (5) they can be reduced to one equation:

$$\left(kAG - EI_x\frac{\partial^2}{\partial Z^2}\right)\left[\rho A\frac{\partial^2 w}{\partial T^2} + F_a(\omega, T)\frac{\partial^2 w}{\partial Z^2}\right] + kAGEI_x\left(1 + v\frac{\partial}{\partial T}\right)\frac{\partial^4 w}{\partial Z^4} = 0. \tag{7}$$

The following parameters can be used to non-dimensionalize equation (7)

$$Z = z\ell, \quad T = k_t t, \quad k_t = \ell^2\sqrt{\frac{\rho A}{EI_x}}, \quad 2\varsigma = \frac{v}{k_t}, \quad e^2 = \frac{EI_x}{kAG\ell^2}, \quad f_o + f(\omega, t) = \frac{F_a(\omega, T)\ell^2}{EI_x}. \tag{8}$$

Substituting relations (8) into Eq. (7) gives:

$$\left(1 - e^2\frac{\partial^2}{\partial z^2}\right)\left[\frac{\partial^2 w}{\partial t^2} + (f_o + f(\omega, t))\frac{\partial^2 w}{\partial z^2}\right] + 2\varsigma\frac{\partial^5 w}{\partial t\,\partial z^4} + \frac{\partial^4 w}{\partial z^4} = 0. \tag{9}$$

Boundary conditions corresponding to simply supported edges written in terms of transverse displacement are as follows:

$$\left.\begin{matrix} z = 0 \\ z = 1 \end{matrix}\right\} \quad w = 0, \quad \frac{\partial^2 w}{\partial z^2} = 0. \tag{10}$$

3. Instability analysis

With the purpose of applying the Liapunov method, we can construct the functional by means of the Parks–Pritchard's method (1969). Thus, let us write the Eq. (9) in formal form $\mathscr{L}w = 0$ and introduce linear operator:

$$\mathscr{N}(\;) = 2\left(1 - e^2\frac{\partial^2}{\partial z^2}\right)\frac{\partial(\;)}{\partial t} + 2\varsigma\frac{\partial^4(\;)}{\partial z^4} \tag{11}$$

which is a formal derivative of the operator $\mathscr{L}$ with respect to $\partial/\partial t$.

Integrating the product $\mathscr{L}w\mathscr{N}w$ on rectangular $C = \Omega \times \Delta = [z{:}0 \leqslant z \leqslant 1] \times [\tau{:}0 \leqslant \tau \leqslant t]$, it is clear

$$\int_0^t \int_0^1 \mathscr{L}w\mathscr{N}w\,\mathrm{d}z\,\mathrm{d}\tau = 0. \tag{12}$$

After applying the partial integration to Eq. (12), the sum of two integrals may be obtained. In the first, integration is only on spatial domain and we choose it to be the Liapunov functional

$$V = \int_0^1 \left\{ \left(\frac{\partial w}{\partial t} - e^2\frac{\partial^3 w}{\partial^2 z\,\partial t} + \varsigma\frac{\partial^4 w}{\partial z^4}\right)^2 + \zeta^2\left(\frac{\partial^4 w}{\partial z^4}\right)^2 + \left(\frac{\partial^2 w}{\partial z^2}\right)^2 + e^2\left(\frac{\partial^3 w}{\partial z^3}\right)^2 - f_\mathrm{o}\left[\left(\frac{\partial w}{\partial z}\right)^2 + 2e^2\left(\frac{\partial^2 w}{\partial z^2}\right)^2 + e^4\left(\frac{\partial^3 w}{\partial z^3}\right)^2\right]\right\}\mathrm{d}z. \tag{13}$$

Since it is evident

$$V|_0^t - \int_0^t \frac{\mathrm{d}V}{\mathrm{d}t}\,\mathrm{d}t = 0 \tag{14}$$

then the second integral in Eq. (14) is a time derivative of the functional (13) along the Eq. (9)

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -\int_0^1 \left[2\varsigma\frac{\partial^5 w}{\partial z^4\,\partial t}\left(\frac{\partial w}{\partial t} - e^2\frac{\partial^3 w}{\partial z^2\,\partial t}\right)^2 + 2\varsigma\left(\frac{\partial^4 w}{\partial z^4}\right)^2 + 2\varsigma f_\mathrm{o}\frac{\partial^4 w}{\partial z^4}\left(\frac{\partial^2 w}{\partial z^2} - e^2\frac{\partial^4 w}{\partial z^4}\right) + 2f(t)\left(\frac{\partial^2 w}{\partial z^2} - e^2\frac{\partial^4 w}{\partial z^4}\right)\left(\frac{\partial w}{\partial t} - e^2\frac{\partial^3 w}{\partial z^2\,\partial t} + \varsigma\frac{\partial^4 w}{\partial z^4}\right)\right]\mathrm{d}z. \tag{15}$$

Functional (13) is positive definite, for normal modes $W_m(z) = \sin\alpha_m z$, $\alpha_m = m\pi$, if

$$f_\mathrm{o} < \frac{\alpha_m^2}{1 + e^2\alpha_m^2}, \quad (m = 1, 2, 3, \ldots). \tag{16}$$

The lowest value of the expression on the right side of the relation (16) is for $m = 1$, and the form of the expression is

$$f_\mathrm{o} < \frac{\pi^2}{1 + e^2\pi^2}, \tag{17}$$

which represents a condition that static loading is smaller than Timoshenko's critical force (1936).

Let a scalar function $\lambda(t)$ be defined as

$$\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}t} \geqslant \lambda(t) \tag{18}$$

for all w and $v = \partial w/\partial t$ satisfying the boundary conditions (10). As a minimum point is a particular case of the stationary point, we may write

$$\delta(\dot{V} - \lambda V) = 0. \tag{19}$$

By using the related Euler's equations we obtain:

$$\begin{aligned}
&\lambda\left[v - e^2\frac{\partial^2 v}{\partial z^2} + \varsigma\frac{\partial^4 w}{\partial z^4} - e^2\left(\frac{\partial^2 v}{\partial z^2} - e^2\frac{\partial^4 v}{\partial z^4} + \varsigma\frac{\partial^6 w}{\partial z^6}\right)\right] + 2\varsigma\left(\frac{\partial^4 v}{\partial z^4} - e^2\frac{\partial^6 v}{\partial z^6}\right)\\
&\quad + f(t)\left(\frac{\partial^2 w}{\partial z^2} - 2e^2\frac{\partial^4 w}{\partial z^4} + e^4\frac{\partial^6 w}{\partial z^6}\right) = 0,\\
&\lambda\left[\varsigma\left(\frac{\partial^4 v}{\partial z^4} - e^2\frac{\partial^6 v}{\partial z^6} + 2\varsigma\frac{\partial^8 w}{\partial z^8}\right) + \frac{\partial^4 w}{\partial z^4} - e^2\frac{\partial^6 w}{\partial z^6} + f_\mathrm{o}\left(\frac{\partial^2 w}{\partial z^2} - 2e^2\frac{\partial^4 w}{\partial z^4} + e^4\frac{\partial^6 w}{\partial z^6}\right)\right]\\
&\quad + \varsigma\left[2\frac{\partial^8 w}{\partial z^8} + 2f_\mathrm{o}\left(\frac{\partial^6 w}{\partial z^6} - e^2\frac{\partial^8 w}{\partial z^8}\right) + f(t)\left(\frac{\partial^6 w}{\partial z^6} - e^2\frac{\partial^8 w}{\partial z^8}\right)\right]\\
&\quad + f(t)\left[\left(\frac{\partial^2 v}{\partial z^2} - e^2\frac{\partial^4 v}{\partial z^4} + \varsigma\frac{\partial^6 w}{\partial z^6}\right) - e^2\left(\frac{\partial^4 v}{\partial z^4} - e^2\frac{\partial^6 v}{\partial z^6} + \varsigma\frac{\partial^8 w}{\partial z^8}\right)\right] = 0.
\end{aligned} \tag{20}$$

By introducing differential operators

$$\begin{aligned}
L_1 &= -\left[\lambda\varsigma\frac{\partial^4}{\partial z^4} + f(t)\frac{\partial^2}{\partial z^2}\left(1 - e^2\frac{\partial^2}{\partial z^2}\right)\right],\\
L_2 &= \lambda\left(1 - e^2\frac{\partial^2}{\partial z^2}\right) + 2\varsigma\frac{\partial^4}{\partial z^4},
\end{aligned} \tag{21}$$

and by applying L_1 to the first and L_2 to the second equation of the system (20), after the addition only one equation is obtained:

$$\begin{aligned}
&-\left[\lambda\varsigma\frac{\partial^4}{\partial z^4} + f(t)\frac{\partial^2}{\partial z^2}\left(1 - e^2\frac{\partial^2}{\partial z^2}\right)\right]\left[\lambda\varsigma\frac{\partial^4}{\partial z^4}\left(1 - e^2\frac{\partial^2}{\partial z^2}\right)w + f(t)\frac{\partial^2}{\partial z^2}\left(1 - e^2\frac{\partial^2}{\partial z^2}\right)^{(2)} w\right]\\
&\quad + \left[\lambda\left(1 - e^2\frac{\partial^2}{\partial z^2}\right) + 2\varsigma\frac{\partial^4}{\partial z^4}\right]\left\{\lambda\left[2\varsigma^2\frac{\partial^8 w}{\partial z^8} + \frac{\partial^4}{\partial z^4}\left(1 - e^2\frac{\partial^2}{\partial z^2}\right)w + f_\mathrm{o}\frac{\partial^2}{\partial z^2}\left(1 - e^2\frac{\partial^2}{\partial z^2}\right)^{(2)} w\right]\right.\\
&\qquad \left. + 2\varsigma\left[\frac{\partial^8 w}{\partial z^8} + f_\mathrm{o}\left(1 - e^2\frac{\partial^2}{\partial z^2}\right)\frac{\partial^6 w}{\partial z^6} + f(t)\left(1 - e^2\frac{\partial^2}{\partial z^2}\right)\frac{\partial^6 w}{\partial z^6}\right]\right\} = 0.
\end{aligned} \tag{22}$$

According to the boundary condition (10) we may write the solution in the form

$$w(z,t) = \sum_{m=1}^{\infty} T_m(t)\sin\alpha_m z \tag{23}$$

and from Eq. (22) we obtain the algebraic equation of the second order:

$$A_m\lambda_m^2 + 2B_m\lambda_m + C_m = 0, \tag{24}$$

where

$$\begin{aligned} A_m &= \left(1+e^2\alpha_m^2\right)\left\{\varsigma^2\alpha_m^8+\alpha_m^2\left[\alpha_m^2-f_o\left(1+e^2\alpha_m^2\right)\right]\left(1+e^2\alpha_m^2\right)\right\},\\ B_m &= 2\varsigma\alpha_m^4\left\{\varsigma^2\alpha_m^8+\alpha_m^2\left[\alpha_m^2-f_o\left(1+e^2\alpha_m^2\right)\right]\left(1+e^2\alpha_m^2\right)\right\},\\ C_m &= 4\varsigma^2\alpha_m^4\left[\alpha_m^8-f_o\alpha_m^6\left(1+e^2\alpha_m^2\right)-f(t)\alpha_m^6\left(1+e^2\alpha_m^2\right)\right]-f^2(t)\alpha_m^4\left(1+e^2\alpha_m^2\right)^3. \end{aligned} \tag{25}$$

Hence, from Eq. (24)

$$\lambda_m = -\frac{2\varsigma\alpha_m^4}{1+e^2\alpha_m^2}+\frac{\alpha_m}{1+e^2\alpha_m^2}\frac{\left|2\varsigma^2\alpha_m^6+f(t)\left(1+e^2\alpha_m^2\right)\right|}{\sqrt{\varsigma^2\alpha_m^6+\left[\alpha_m^2-f_o\left(1+e^2\alpha_m^2\right)\right]\left(1+e^2\alpha_m^2\right)}}. \tag{26}$$

By solving the differential inequality (18), we obtain the following estimation of the functional:

$$V(t) \geqslant V(0)\exp\left[t\left(\frac{1}{t}\int_0^t \lambda(\tau)\,d\tau\right)\right]. \tag{27}$$

Therefore, it can be stated that the trivial solution of Eq. (9) is almost surely asymptotically instable if

$$\lim_{t\to\infty}\frac{1}{t}\int_0^t \lambda(\tau)\,d\tau \geqslant 0, \tag{28}$$

or, when the process $f(t)$ is ergodic and stationary:

$$E\{\lambda(t)\} \geqslant 0, \tag{29}$$

where E denotes the operator of mathematical expectation, and

$$\lambda(t) = \min_m \lambda_m(t). \tag{30}$$

4. Numerical results and discussion

The expression (26) and inequalities (28) or (29) give the possibility to obtain critical retardation time, guaranteeing an almost sure asymptotic instability as a function of the statistic characteristics of loading. The almost sure asymptotic instability region is defined as a set where the retardation time is smaller then it's critical value. By applying Schwartz's inequality to the relation (30) one obtains

$$\sigma^2 \geqslant \frac{4\varsigma^2\alpha_m^6\left[\alpha_m^2-f_o\left(1+e^2\alpha_m^2\right)\right]}{\left(1+e^2\alpha_m^2\right)^3}. \tag{31}$$

If probability density function $p(f)$ is known for the process $f(t)$, one can obtain more precise results, than the ones obtained from relation (31). The boundaries of the almost sure instability are calculated by using the corresponding Gauss–Christoffel quadratures in Eq. (29)

$$\int_R \lambda(f)p(f)\,df \approx \sum_{k=1}^n W_k\lambda(\xi_k). \tag{32}$$

For Gaussian process:

$$R = \{f : -\infty < f < +\infty\}, \quad p(f) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-f^2/(2\sigma^2)\right), \tag{33}$$

we take the parameters of Gauss–Hermite quadrature where ξ_k are zeros of Hermite polynomial H_n, $(k = 1, 2, \ldots, n)$, and

$$W_k = \frac{2^{n+1}\sqrt{\pi}n!}{\left[H'_{n+1}(\xi_k)\right]^2}. \tag{34}$$

For the harmonic process we set $f(t) = A\cos(\omega t + \theta)$, where A and ω are fixed amplitude and frequency, and θ is a uniformly distributed random phase on the interval $[0, 2\pi)$. In order to compare both processes the variance of harmonic process $\sigma^2 = A^2/2$ is used. Then:

$$R = \left\{f : -\sigma\sqrt{2} < f < +\sigma\sqrt{2}\right\}, \quad p(f) = \frac{1}{\pi\sqrt{2\sigma^2 - f^2}}, \tag{35}$$

and we take the Gauss–Chebyshev quadrature, whose parameters are:

$$\xi_k = \cos\frac{2k-1}{2n}\pi, \qquad W_k = \frac{\pi}{n}, \quad (k = 1, 2, \ldots, n). \tag{36}$$

Numerical results are shown in Figs. 2–4. Boundaries of the almost sure instability are presented with full line for Gaussian process and dashed line for harmonical process.

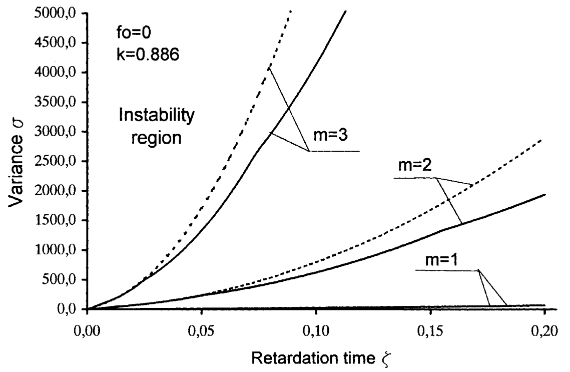

Fig. 2. Influence of the mode number on the instability regions.

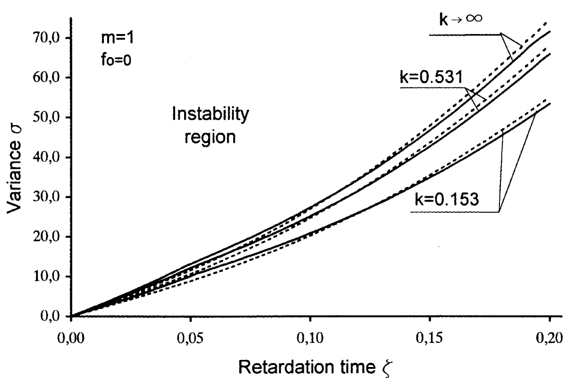

Fig. 3. Influence of the section shape factor on the instability regions.

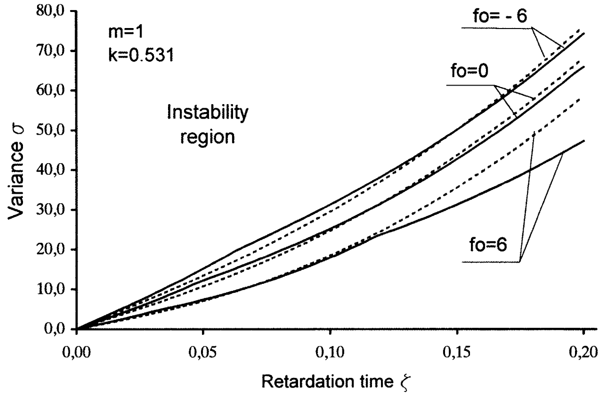

Fig. 4. Influence of the statical force on the instability regions.

In Fig. 2 instability regions for the first three modes ($m = 1, 2, 3$) are plotted. The difference between boundaries for the first mode ($m = 1$), is very small for both processes, but for higher modes regions of instability for Gaussian process are greater. In Fig. 3 regions of instability as a function of the section shape factor are given. As an example we take a beam with $E/G = 2.6$ and $\ell/r = 30$, where $r = (I_x/A)^{1/2}$ is the radius of gyration of the cross-section, according to Banerjee and Williams (1994). Numerical calculation is performed for the next five cases: the effect of transverse shear is neglected ($k \to \infty$), solid circular cross-section ($k = 0.886$), solid rectangular ($k = 0.850$), thin circular ($k = 0.531$) and thin rectangular, $b = 3d$, ($k = 0.153$). In the cases of solid circular ($k = 0.886$) and solid rectangular ($k = 0.850$) boundaries of the instability regions are not shown, because they are very close to limit curve $k \to \infty$. According to this, neglecting the transverse shear in those cases is justified. In cases of circular and rectangular cross-sections ($k = 0.531$ and $k = 0.153$), it is evident that neglecting the transverse shear will be an error. Generally speaking, the influence of transverse shear leads to a growth of the almost sure regions of instability. The regions of instability decrease when statical force is changed from compression ($f_o = 6$) to tension ($f_o = -6$), Fig. 4.

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